Master QFin, CTFI
Minitest 1 Tue 17.5.2016, 9.15

Hints

- This is a closed-book exam; no pocket calculators etc.
- Please mark every sheet with your name and Mat.Nr.
- Good luck !

1. Brownian motion

a) (3 points) Give the definition of standard Brownian motion.

b) (3 points) Let W be standard Brownian motion. Show that for fixed $\alpha > 0$ the process $B_t = \alpha W_{\frac{t}{\alpha^2}}$, $\quad t \geq 0$ is a Brownian motion.

2. Let W_t be standard Brownian motion and define $\mathcal{F}_t := \sigma(W_s, s \leq t)$.

a) (2 points) Show that for $t > s$,

$$E(W_t^2 - W_s^2 \mid \mathcal{F}_s) = E\big((W_t - W_s)^2 \mid \mathcal{F}_s\big)$$

b) (2 points) Use the result from a) to show that $W_t^2 - t$ is a martingale with respect to the filtration $\{\mathcal{F}_t\}$.

3. Stopping times Consider a probability space $(\Omega, \mathcal{F}, P)$ with filtration $\{\mathcal{F}_t\}$.

a) (2 points) Give the definition of a stopping time.

b) (2 points) Let τ be a stopping time and $s > 0$ be deterministic. Show that $\tau + s$ is a stopping time.

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Master QFin, CTFI
Minitest 2 Tue 31.5.2016, 9.15

Hints

- This is a closed-book exam; no pocket calculators etc.
- Please mark every sheet with your name and Mat.Nr.
- Good luck !

1. (3 points) Let B be Brownian motion. Use Itos formula to compute the semimartingale decomposition of $X_t = B_t^3$ and compute $[X]_t$

2. Geometric Brownian motion Suppose that the process S satisfies the SDE $dS_t = \mu S_t dt + \sigma S_t dW_t$ for constants $\mu \in \mathbb{R}$, $\sigma > 0$ with initial value $S_0 > 0$.

a) (2 points) Use the Ito formula to compute the dynamics of $\ln S_t$. (You may assume that $S_t > 0$ for all t).

b) (2 points) Use the result from a) to solve the SDE for S.

3. (2 points) Consider a twice differentiable function F on $\mathbb{R}$ and a Brownian motion W. Use the Ito formula to show that $F(W_t) - \frac{1}{2}\int_0^t F''(W_s)ds$ is a (local) martingale.

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Master QFin, CTFI
Minitest 3 Tue 14.6.2016, 9.15

Hints

- This is a closed-book exam.
- Please mark every sheet with your name and Mat.Nr.
- Good luck !

1. Feynman Kac (5 points) Use the Feynman Kac formula to solve the PDE

$$f_t(t,x) + \frac{1}{2}\sigma^2 x^2 f_{xx} = 0,\ (t,x) \in [0,T) \times \mathbb{R},$$

with terminal condition $f(T,x) = 1_{\{x>1\}}$.

2. Options (4 points) Construct a delta neutral hedging strategy for a put option with strike $K = 40$ and $T = 30$ days, on 10 units of stock with parameters $S_0 = 40$ and $\sigma = 0.2$. Assume that the interest rate equals $r = 0.02$ and that the year has 360 days. (Hint: price of a put in the Black Scholes model is given by $P_{BS}(t,S;\sigma,r,K,T) = -S_t N(d_1) + Ke^{-r(T-t)}N(-d_2)$, d_1 and d_2 as for a call.)

3. Greeks (3 points) Use put-call parity to show that the Vega of a call and of a put option with identical strike K and maturity date T on the same stock S are the same.